Number Theory 16

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Q) Show that $(a+b)^{p^i} \equiv a^{p^i} + b^{p^i} \pmod p$ if p is a prime and i is any non-negative integer

Ans:— $(a+b)^{p^i} \to$ True for $i=1$

Let it be true for $i=n$, i.e., $(a+b)^{p^n} \equiv \left(a^{p^n} + b^{p^n}\right) \pmod p$

$(a+b)^{p^{n+1}} = (a+b)^{p^n \cdot p} = \left((a+b)^p\right)^{p^n}$

$$= \underbrace{(a+b)^p (a+b)^p \cdots (a+b)^p}_{p^n}$$

$xy \pmod p$

$\equiv \left(x \pmod p\right)\left(y \pmod p\right)$

$$\equiv (a^p + b^p)(a^p + b^p) \cdots (a^p + b^p) \pmod p$$

$$\equiv (a^p + b^p)^{p^n} \pmod p$$

$$\equiv (a^p)^{p^n} + (b^p)^{p^n} \pmod p$$

$$\equiv a^{p^{n+1}} + b^{p^{n+1}} \pmod p$$

Q) Suppose $a, b, d \in \mathbb{Z}$ and $n \in \mathbb{N}$ such that $ad \equiv bd \pmod n$

Show that,

$$a \equiv b \left(\bmod \frac{n}{\gcd(n,d)}\right)$$

$p \Rightarrow q$ } not equivalent
$q \Rightarrow p$

Ans:— $\gcd(n,d) = g \Rightarrow n = gq_1, \; d = gq_2$ $\frac{n}{\gcd(n,d)} = q_1$

$\gcd(q_1, q_2) = 1$

$ad = nk_1 + bd$

$\Rightarrow (a-b)d = nk_1$

$\Rightarrow a - b = \frac{nk_1}{d}$

$\Rightarrow a = b + \frac{nk_1}{d}$

$a = b + \frac{gq_1k_1}{gq_2} \Rightarrow a = b + q_1 \underbrace{\frac{k_1}{q_2}}_{\in \mathbb{Z}}$

$\Rightarrow a = b + q_1 c \quad [c \in \mathbb{Z}]$

$\Rightarrow a \equiv b \pmod{q_1}$

Q) 3 Bricks $\to$ $5 \times 4.5 \times 3$

How many different heights can you build up using them?

Ans:- $5x + 4.5y + 3z = h$ where $x+y+z = 3$

Q) 5 digit number is floppy if $(a_1 a_2 a_3 a_4 a_5) = N$ $\Rightarrow a_4 \times a_5 = 32$, $\sum_{i=1}^{5} a_i = 36$

$x = \dfrac{\text{no. of floppy numbers}}{\text{no. of floppy numbers divisible by } 36}$

Then find x

Ans:- $(a_4, a_5) \in \{(4,8), (8,4)\}$

$\sum a_i = 36 \Rightarrow a_1 + a_2 + a_3 = 24$

$\Rightarrow (a_1 a_2 a_3 a_4 \times 100) + (a_4 a_5)$

as $a_4 a_5 = 48$ or 84

$4 \mid N$ and $9 \mid 36$ as $9 \mid 36$

$\Rightarrow 36 \mid N \Rightarrow x = 1$

6	9	9
7	8	9
7	9	8
8	7	9
8	8	8
8	9	7
9	6	9
9	7	8
9	8	7
9	9	6

no need for these cases

HomeWork

Show that for any fixed integer $n \geq 1$, the sequence,

$$2, 2^2, 2^{2^2}, 2^{2^{2^2}}, \ldots \pmod{n}$$

is eventually constant

$(\ldots c_1, c_n \ldots\ldots a, a, a, \ldots a, \ldots \text{upto } \infty \pmod{n})$

HomeWork

HomeWork

$34! = 2952327990 39a0414084761860964 35b0000000$

Then find a and b